Laws of Motion

Fill in the Blanks

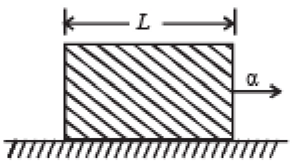
Q.1. A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0.6. If the acceleration of the truck is 5 m/s^2, the frictional force acting on the block is ..... newtons. (1984 - 2 Marks)

Ans. 5

Solution. As seen by the observer on the ground, the frictional force is responsible to move the mass with an acceleration of 5 m/s^2.

Therefore, frictional force = m × a = 1 × 5 = 5 N.

Q.2. A uniform rod of length L and density ρ is being pulled along a smooth floor with a horizontal acceleration α (see Fig.) The magnitude of the stress at the transverse crosssection through the mid- point of the rod is .... (1993 - 1 Mark)



Ans. $\rho L\alpha/2$

Solution. Let A be the area of cross-section of the rod.

Consider the back half portion of the rod.

Mass of half portion of the rod = $\rho AL/2$

The force responsible for its acceleration is

$$f = \frac{\rho AL}{2} \times \alpha \qquad \therefore \text{ Stress} = \frac{f}{A} = \frac{\rho L\alpha}{2}$$

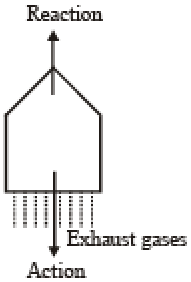
True/False

Q.1. A rocket moves forward by pushing the surrounding air backwards.

Ans. F

Solution. KEY CONCEPT : The rocket moves forward when the exhaust gases are thrown backward.

Here exhaust gases thrown backwards is action and rocket moving forward is reaction.



Note : This phenomenon takes place in the absence of air as well.

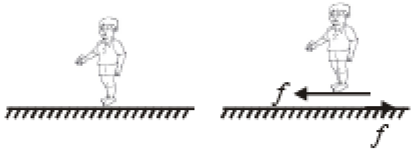
Q.2. When a person walks on a rough surface, the frictional force exerted by the surface on the person is opposite to the direction of his motion. (1981 - 2 Marks)

Ans. F

Solution. KEY CONCEPT : Friction force opposes the relative motion of the surface of contact.

When a person walks on a rough surface, the foot is the surface of contact. When he pushes the foot backward, the motion of surface of contact tends to be backwards.

Therefore the frictional force will act forward (in the direction of motion of the person)



Q.3. A simple pendulum with a bob of mass m swings with an angular amplitude of 40°. When its angular displacement is 20°, the tension in the string is greater than mg cos 20°. (1984 - 2 Marks)

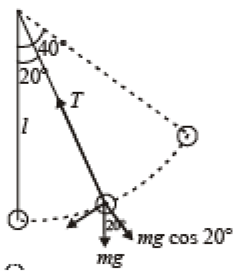
Ans. T

Solution. As the angular amplitude of the pendulum is 40°, the bob will be in the mid of the equilibrium position and the extreme position as shown in the figure

Note : For equilibrium of the bob, T – mg cos 20° =

$= \frac{mv^2}{l}$,

where l is the length of the pendulum and is the velocity of the bob.

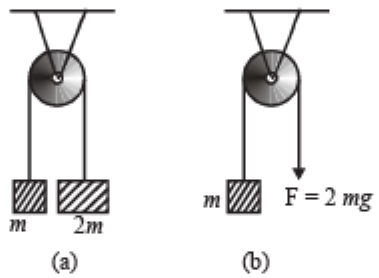


$\therefore \quad T = mg \cos 20° + \frac{mv^2}{l}$

$\frac{mv^2}{l}$ is always a positive quantity.

Hence, $T > mg \cos 20°$.

Q.4. The pulley arrangements of Figs. (a) and (b) are identical. The mass of the rope is negligible. In (a) the mass m is lifted up by attaching a mass 2m to the other end of the rope. In (b) , m is lifted up by pulling the other end of the rope with a constant downward force F = 2 mg. The acceleration of m is the same in both cases (1984 - 2 Marks)



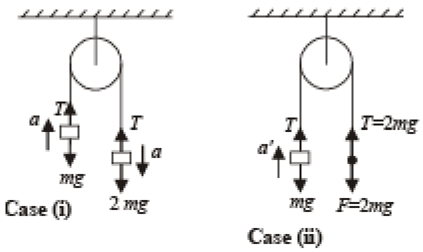
Ans. F

Solution. Case (i) For mass m

T – mg = ma ... (i)

For mass 2m

2mg – T = 2ma ... (ii)



From (i) and (ii) a = g/3

Case (ii) T – mg = ma'

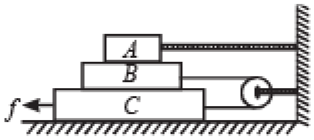
2mg – mg = ma' [∴ T = 2mg]

∴ a' = g

Hence, a < a'

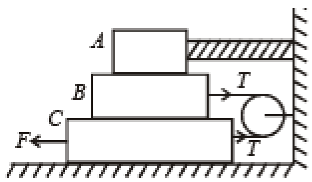
Subjective Questions

Q.1. In the diagram shown, the blocks A, B and C weight, 3 kg, 4 kg and 5 kg respectively. The coefficient of sliding friction between any two surface is 0.25. A is held at rest by a massless rigid rod fixed to the wall while B and C are connected by a light flexible cord passing around a frictionless pulley. Find the force F necessary to drag C along the horizontal surface to the left at constant speed. Assume that the arrangement shown in the diagram, B on C and A on B, is maintained all through. ($g = 9.8\ m/s^2$)

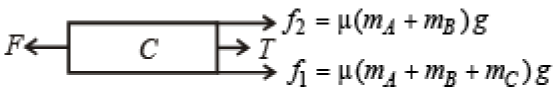


Ans. 71.05 N

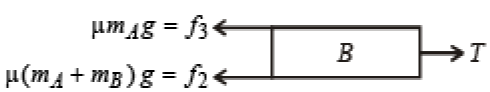
Solution. When force F is applied on C, the block C will move towards left.



The F.B.D. for mass C is



As C is moving with constant speed $F = f_1 + f_2 + T$ ... (i) F.B.D. for mass B is



As B is moving with constant speed $f_2 + f_3 = T$ (ii) Subtracting (ii) from (i)

$F - (f_2 + f_3) = f_1 + f_2 + T - T = f_1 + f_2$

$\Rightarrow F = f_1 + 2f_2 + f_3 = \mu\ (m_A + m_B + m_C)\ g + 2\mu\ (m_A + m_B)\ g + \mu\ m_A g$

$F = \mu\ (4\ m_A + 3\ m_B + m_C)\ g$

$= 0.25\ [4 \times 3 + 3 \times 4 + 5] \times 9.8 = 71.05$ N

Q.2. Two cubes of masses m_1 and m_2 be on two frictionless slopes of block A which rests on a horizontal table. The cubes are connected by a string which passes over a pulley as shown in the figure.

To what horizontal acceleration f should the whole system (that is blocks and cubes) be subjected so that the cubes do not slide down the planes.

What is the tension of the string in this situation? (1978)

Ans.

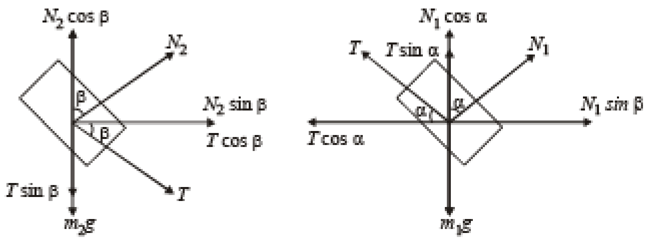
$$f = \frac{(m_1 \sin\alpha + m_2 \sin\beta)\ g}{m_1 \cos\alpha + m_2 \cos\beta};\ T = \frac{m_1 m_2\ g\ \sin(\alpha - \beta)}{m_1 \cos\alpha + m_2 \cos\beta}$$

Solution. Without Pseudo Force

F.B.D for mass m_2

$N_2 \cos\beta = T \sin\beta + m_2 g$ ... (i)

and $(N_2 \sin\beta + T \cos\beta) = m_2 f$ ... (ii)



FBD for mass m_1

$N_1 \cos\alpha + T \sin\alpha = m_1 g$ ... (iii)

and $(N_1 \sin\alpha - T \cos\alpha) = m_1 f$ ... (iv)

On solving the four equations, we get the above results.

Q.3. A horizontal uniform rope of length L, resting on a frictionless horizontal surface, is pulled at one end by force F. What is the tension in the rope at a distance l from the end where the force is applied? (1978)

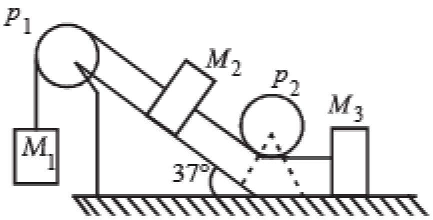
Ans. $T = F\left(1 - \frac{\ell}{L}\right)$

Solution.

From equation (i) $T = \frac{M}{L}(L - \ell)a$

Also, $F = Ma$ $\quad \therefore \frac{T}{F} = \left(\frac{L-\ell}{L}\right) \Rightarrow T = F\left(1 - \frac{\ell}{L}\right)$

Q.4. Masses M_1, M_2 and M_3 are connected by strings of negligible mass which pass over massless and friction less pulleys P_1 and P_2 as shown in fig The masses move such that the portion of the string between P_1 and P_2 in parallel to the inclined plane and the portion of the string between P_2 and M_3 is horizontal. The masses M_2 and M_3 are 4.0 kg each and the coefficient of kinetic friction between the masses and the surfaces is 0.25. The inclined plane makes an angle of 37° with the horizontal. (1981- 6 Marks)



If the mass M_1 moves downwards with a uniform velocity, find

(i) the mass of M_1

(ii) The tension in the horizontal portion of the string

$(g = 9.8 \text{ m/sec}^2, \sin 37° \simeq 3/5)$

Ans. 4.2 Kg, 9.8 N

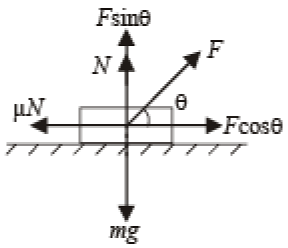
Solution. (a) If M_1, M_2 and M_3 are considered as a system, then the force responsible to more them is M_1g and the retarding force is $(M_2g \sin\theta + \mu M_2g \cos\theta + \mu M_3g)$. These two should be equal as the system is moving with constant velocity.

Q.5. A particle of mass m rests on a horizontal floor with which it has a coefficient of static friction μ . It is desired to make the body move by applying the minimum possible force F. Find the maguitude of F and the direction in which it has to be applied. (1987 - 7 Marks)

Ans. mgsinθ, $\tan^{-1}\mu$

Solution. Let F be the force applied to move the body at an angle θ to the horizontal.

The body will move when



$F \cos\theta = \mu N$ ... (i)

Applying equilibrium of forces in the vertical direction we get

$F \sin\theta + N = mg$

$\Rightarrow N = mg - F \sin\theta$ ... (ii)

⇒ From (i) and (ii)

$$F = \frac{\mu mg}{\cos\theta + \mu \sin\theta} \text{ ... (iii)}$$

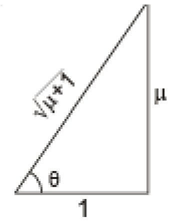
Differentiating the above equation w.r.t. θ , we get

$$\frac{dF}{d\theta} = \frac{\mu mg}{(\cos\theta + \mu\sin\theta)^2}[-\sin\theta + \mu\cos\theta] = 0$$

$\Rightarrow \theta = \tan^{-1}\mu$

This is the angle for minimum force.

To find the minimum force substituting these values in equation (iii)



$$\sin\theta = \frac{\mu}{\sqrt{\mu^2+1}}, \quad \cos\theta = \frac{1}{\sqrt{\mu^2+1}}$$

$$F = \frac{\mu mg}{\frac{1}{\sqrt{\mu^2+1}} + \frac{\mu}{\sqrt{\mu^2+1}} \times \mu}$$

$$\Rightarrow \quad F = \frac{\mu mg(\sqrt{\mu^2+1})}{\mu^2+1} = \frac{\mu mg}{\sqrt{\mu^2+1}}$$

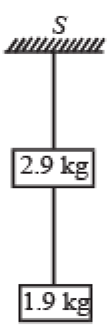
$$\Rightarrow \quad F = mg\sin\theta$$

Q.6. Two blocks of mass 2.9 kg and 1.9 kg are suspended from a rigid support S by two inextensible wires each of length 1 meter, see fig. The upper wire has negligible mass and the lower wire has a uniform mass of 0.2 kg/m.

The whole system of blocks wires and support have an upward acceleration of 0.2 m/s². Acceleration due to gravity is 9.8 m/s². (1989 - 6 Marks)

(i) Find the tension at the mid-point of the lower wire.

(ii) Find the tension at the mid-point of the upper wire.



Ans. 20N, 50N

Solution. Let λ be the mass per unit length of lower wire.

Let us consider the dotted portion as a system and the tension T accelerates the system upwards

$$\therefore \quad T - (m_1 + \lambda \ell) g = (m_1 + \lambda \ell) a$$

$$\therefore \quad T = (m_1 + \lambda \ell)(a + g)$$
$$= (1.9 + 0.2 \times 0.5)(9.8 + 0.2) = 2 \times 10 = 20 N$$

To find tension T ' Let us consider the dotted portion given in figure (2)

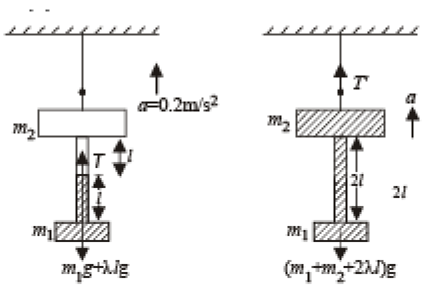


Fig. (1) Fig. (2)

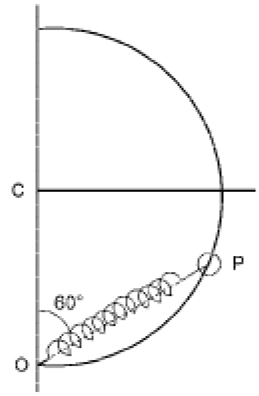
$$T' - (m_2 g + \lambda \times 2 \ell g + m_1 g) = (m_1 + \lambda 2 \ell + m_2) a$$

$$\therefore \quad T' = (m_1 + \lambda 2 \ell + m_2)(a + g)$$
$$= (1.9 + 0.2 \times 1 + 2.9)(10) = 5 \times 10 = 50 N.$$

Alternatively considering m_1, m_2 and lower wire as a system T' – 5 g = 5 a

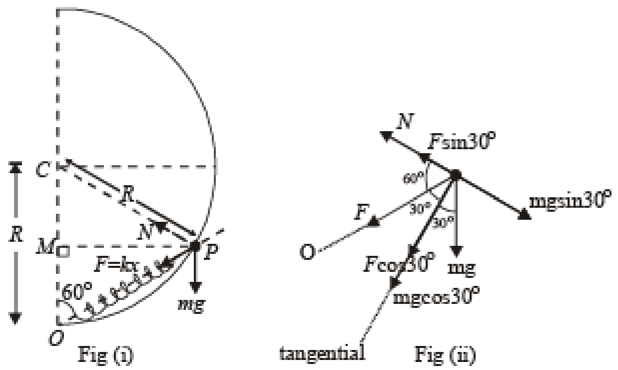
Q.7. A smooth semicircular wire-track of radius R is fixed in a vertical plane. One end of a massless spring of natural length 3R/4 is attached to the lowest point O of the wire-track. A small ring of mass m, which can slide on the track, is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle of 60° with the vertical. The spring constant K= mg/R.

Consider the instant when the ring is released, and (i) draw the free body diagram of the ring, (ii) determine the tangential acceleration of the ring and the normal reaction. (1996 - 5 Marks)



Ans. $\frac{5\sqrt{3}}{8}g, \frac{3mg}{8}$

Solution.



In DOCP, OC = CP = R

∴ ∠COP = ∠CPO = 60° ⇒ ∠OCP = 60°

∴ ΔOCP is an equilateral triangle ⇒ OP = R

∴ Extension of string $= R - \frac{3R}{4} = \frac{R}{4} = x$

The forces acting are shown in the figure (i)

The free body diagram of the ring is shown in fig. (ii)

Force in the tangential direction

= F cos 30° + mg cos 30

= [kx + mg] cos 30°

$$F_t = \frac{5mg}{8}\sqrt{3} \quad \therefore F_t = ma_t \quad \Rightarrow \quad a_t = \frac{5\sqrt{3}}{8}g$$

Also, when the ring is just released

N + F sin 30° = mg sin 30°

$$\Rightarrow \quad N = (mg - F)\sin 30° = \left(mg - \frac{mg}{4}\right) \times \frac{1}{2} = \frac{3mg}{8}$$

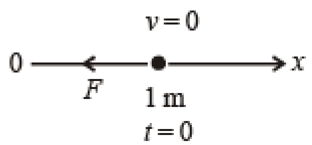
Q.8. A particle of mass 10^{-2} kg is moving along the positive x axis under the influence of a force F(x) = – K/(2x²) where K = 10^{-2} N m². At time t = 0 it is at x = 1.0 m and its velocity is v = 0. (1998 - 8 Marks)

(a) Find its velocity when it reaches x = 0.50 m.

(b) Find the time at which it reaches x = 0.25 m.

Ans. (a) $-1\,\text{m/s}$ (b) $\left(\frac{\pi}{3} + \frac{\sqrt{3}}{4}\right)$ sec

Solution. m = 10^{-2} kg, motion is along positive X-axis



$$F(x) = -\frac{K}{2x^2}, K = 10^{-2}\,Nm^2; \text{At } t = 0, x = 1.0\,\text{m}$$
and $V = 0$

(a) $F(x) = \frac{-K}{2x^2}$ or $m\left(\frac{dV}{dx}\right)V = -\frac{K}{2x^2}$

or $m\int_0^v V\,dV = -\int_1^x \frac{K}{2x^2}dx$

or $\dfrac{mV^2}{2} = \left[\dfrac{K}{2x}\right]_1^x = \dfrac{K}{2}\left(\dfrac{1}{x} - 1\right)$

or $V^2 = \dfrac{K}{m}\left(\dfrac{1}{x} - 1\right)$ or $|\bar{V}| = \pm\sqrt{\dfrac{K}{m}\left(\dfrac{1}{x} - 1\right)}$ ... (i)

Initially the particle was moving in + X direction at x = 1.

When the particle is at x = 0.5, obviously its velocity will be in –X direction. The force acting in –X direction first decreases the speed of the particle, bring it momentarily at rest and then changes the direction of motion of the particle.

When $x = 0.5\,\text{m} : |\bar{V}| = -\sqrt{\dfrac{K}{m}\left(\dfrac{1}{0.5} - 1\right)}$

$= -\sqrt{\dfrac{K}{m}} = -\sqrt{\dfrac{10^{-2}}{10^{-2}}} = -1\,\text{m/s}$

(b) As $\dfrac{K}{m} = 1$ m/s, hence from (i)

$V = \dfrac{dx}{dt} = -\sqrt{\dfrac{1-x}{x}}$

Note : We have chosen –ve sign because force tends to decrease the displacement with time

$\sqrt{\dfrac{x}{1-x}}\,dx = -dt;\ \int_1^{0.25}\sqrt{\dfrac{x}{1-x}}\,dx = \int_0^t -dt$

Put x = $\sin^2\theta$, dx = 2 sin θ cos θ d θ

So, $\displaystyle\int_{\pi/2}^{\pi/6} 2\sin^2\theta\, d\theta = -t$

$\cos 2\theta = 1 - 2\sin^2\theta;\quad 2\sin^2\theta = 1 - \cos 2\theta$

$\displaystyle\int_{\pi/2}^{\pi/6}(1 - \cos 2\theta)\,d\theta = -t;\ \left[\theta - \sin\frac{2\theta}{2}\right]_{\pi/2}^{\pi/6} = -t$

$\dfrac{\pi}{6} - \dfrac{1}{2}\sin\dfrac{\pi}{3} - \dfrac{\pi}{2} - \dfrac{1}{2}\sin\pi = -t$

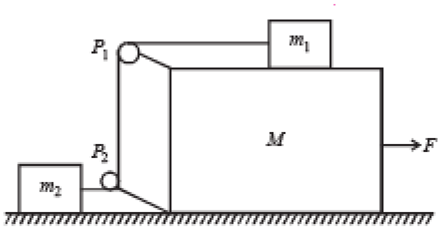
$t = \left(\dfrac{\pi}{3} + \dfrac{\sqrt{3}}{4}\right)$ sec.

Q.9. In the figure masses m_1, m_2 and M are 20 kg, 5 kg and 50 kg respectively. The coefficient of friction between M and ground is zero. The coefficient of friction between m_1 and M and that between m_2 and ground is 0.3. The pulleys and the strings are massless. The string is perfectly horizontal between P_1 and m_1 and also between P_2 and m_2. The string is perfectly vertical between P_1 and P_2. An external horizontal force F is applied to the mass M. Take g = 10 m/s^2 (2000 - 10 Marks)

(a) Draw a free body diagram for mass M, clearly showing all the forces.

(b) Let the magnitude of the force of friction between m1 and M be f_1 and that between m_2 and ground be f_2. For a particular F it is found that $f_1 = 2f_2$. Find f_1 and f_2.

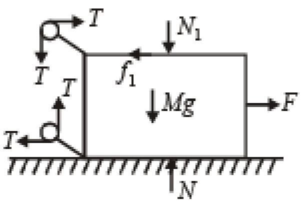
Write equations of motion of all the masses. Find F, tension in the string and acceleration of the masses



Ans. (b) F = 60 N; T = 18 N

Solution. Given m_1 = 20 kg, m_2 = 5 kg, M = 50 kg, μ = 0.3 and g = 10 m/s^2

(A) Free body diagram of mass M is



(B) The maximum value of f_1 is

$(f_1)max = (0.3) (20) (10) = 60$ N

The maximum value of f_2 is $(f_2)max = (0.3) (5) (10) = 15$ N

Forces on m_1 and m_2 in horizontal direction are as follows :

Note : There are only two possibilities.

(1) Either both m_1 and m_2 will remain stationary (w.r.t. ground) or (2) both m_1 and m_2 will move (w.r.t. ground). First case is possible when.

$T \leq (f_1)max$ or $T \leq 60$ N

and $T \leq (f2)max$ or $T \leq 15$ N

These conditions will be satisfied when $T \leq 15$ N say T = 14 then $f_1 = f_2 = 14$ N.

Therefore the condition $f_1 = 2f_2$ will not be satisfied.

Thus m_1 and m_2 both can't remain stationary.

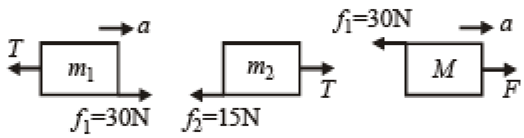
In the second case, when m1 and m2 both move

$f_2 = (f_2)$ max = 15 N

Therefore, $f_1 = 2f_2 = 30$ N

Note : Since $f_1 < (f_1)_{max}$, there is no relative motion between m_1 and M, i.e., all the masses move with same acceleration, say 'a'.

Free body diagrams and equations of motion are as follows:



For m_1 : $30 - T = 20$ a ... (i)

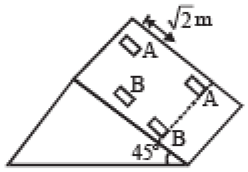
For m_2 : $T - 15 = 5$ a ... (ii)

For M : $F - 30 = 50$ a ... (iii)

Solving these three equations, we get,

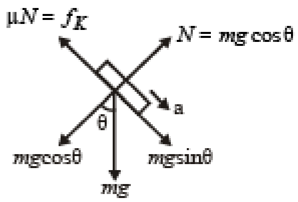
$F = 60\,N,\, T = 18\,N$ and $a = \frac{3}{5}\,m/s^2$.

Q.10. Two block A and B of equal masses are placed on rough inclined plane as shown in figure. When and where will the two blocks come on the same line on the inclined plane if they are released simultaneously? Initially the block A is √2 m behin d th e block B. Co-efficien t of kinetic friction for the blocks A and B are 0.2 and 0.3 respectively ($g = 10\ m/s^2$). (2004 - Marks)



Ans. 8√2m, 7√2 m, 2 sec.

Solution.



$$a = \frac{mg\sin\theta - \mu_k mg\cos\theta}{m}$$

$\therefore a_A = g\sin\theta - \mu_{k,A}\, g\cos\theta$ ... (i)

and $a_B = g\sin\theta - \mu_{k,B}\, g\cos\theta$ ... (ii)

Putting values we get

$a_A = 4\sqrt{2}\ m/s^2$ and $a_B = 3.5\sqrt{2}\ m/s^2$

Let a_{AB} is relative acceleration of A w.r.t. B. Then

$$a_{AB} = a_A - a_B$$
$$L = \sqrt{2}\ m$$

[where L is the relative distance between A and B]

Then $L = \frac{1}{2} a_{AB} t^2$

or $t^2 = \frac{2L}{a_{AB}} = \frac{2L}{a_A - a_B}$

Putting values we get, $t^2 = 4$ or $t = 2s$.

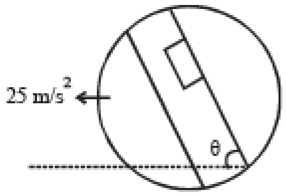
Distance moved by B during that time is given by

$S = \frac{1}{2} a_B t^2 = \frac{1}{2} \times 3.5\sqrt{2} \times 4 = 7\sqrt{2}$ m

Similarly for $A = 8\sqrt{2}$ m

Q.11. A circular disc with a groove along its diameter is placed horizontally on a rough surface. A block of mass 1 kg is placed as shown. The co-efficient of friction between the block and all surfaces of groove and horizontal surface in contact is μ = 2/5. The disc has an acceleration of 25 m/s² towards left. Find the acceleration of the block with respect to disc. (2006 - 6M)

Given $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$.



Ans. 10 m/s²

Solution. Applying pseudo force ma and resolving it.

Applying $F_{net} = ma_r$

$ma \cos\theta - (f_1 + f_2) = ma_r$

$ma \cos\theta - \mu N_1 - \mu N_2 = ma_r$

$ma \cos\theta - \mu ma \sin\theta - \mu mg = ma_r$

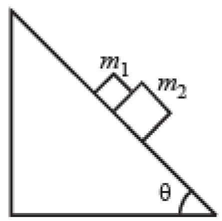
$\Rightarrow a_r = a \cos\theta - \mu a \sin\theta - \mu g$

$$= 25 \times \frac{4}{5} - \frac{2}{5} \times 25 \times \frac{3}{5} - \frac{2}{5} \times 10 = 10\,\text{m/s}^2$$

Match the Following

DIRECTIONS (Q. No. 1) : Following question has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

Q.1. A block of mass m_1 = 1 kg another mass m_2 = 2 kg, are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in List-I. The coefficient of friction between the block m_1 and plane is always zero. The coefficient of static and dynamic friction between the block m2 and the plane are equal to μ = 0.3. In List-II expressions for the friction on block m_2 are given. Match the correct expression of the friction in List-II with the angles given in List-I, and choose the correct option. The acceleration due to gravity is denoted by g.



[Useful information: tan (5.5°) ≈ 0.1; tan (11.5°) ≈ 0.2; tan (16.5°) ≈ 0.3]

	List-I	**List-II**
P.	θ = 5°	$m_2g \sin\theta$
Q.	θ = 10°	$(m_1 + m_2)g \sin\theta$
R.	θ = 15°	$\mu m_2 g \cos\theta$
S.	θ = 20°	$\mu(m_1 + m_2)g \cos\theta$

Code:

(a) P-1, Q-1, R-1, S-3

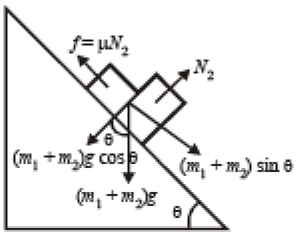
(b) P-2, Q-2, R-2, S-3

(c) P-2, Q-2, R-2, S-4

(d) P-2, Q-2, R-3, S-3

Ans. (d)

Solution. If $(m_1 + m_2) \sin\theta < \mu N_2$ the bodies will be at rest i.e., $(m_1 + m_2)g \sin\theta < \mu m_2 g \cos\theta$



$$\tan\theta < \frac{\mu m_2 g}{(m_1 + m_2)g}$$

$$\Rightarrow \quad \tan\theta < \frac{\mu m_2}{m_1 + m_2}$$

$$\Rightarrow \quad \tan\theta < \frac{0.3 \times 2}{1 + 2}$$

$\Rightarrow \tan\theta < 0.2$ i.e.,

If the angle $\theta < 11.5°$ the frictional force is less than

$\mu N_2 = \mu m_2 g = 0.3 \times 2 \times g = 0.6$ g

and is equal to $(m_1 + m_2)g \sin\theta$

At $\theta = 11.5°$ the bodies are on the verge of moving, f = 0.6 g

At $\theta > 11.5°$ the bodies start moving and f = 0.6 g

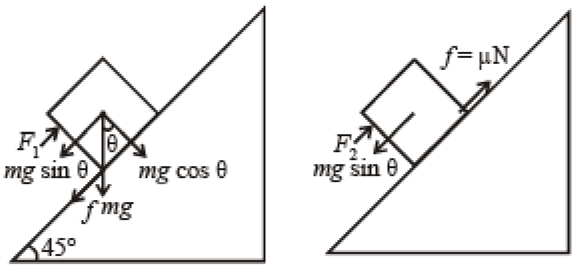
The above relationship is true for (d).

Integer Value Correct Type

Q.1. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define N = 10 μ, then N is (2011)

Ans. 5

Solution.



The pushing force $F_1 = mg \sin\theta + f$

$\therefore F_1 = mg \sin\theta + \mu mg \cos\theta = mg(\sin\theta + \mu \cos\theta)$

The force required to just prevent it from sliding down

$F_2 = mg \sin\theta - \mu N = mg(\sin\theta - \mu \cos\theta)$

Given , $F_1 = 3F_2$

$\therefore \sin\theta + \mu\cos\theta = 3(\sin\theta - \mu\cos\theta)$

$\therefore 1 + \mu = 3(1 - \mu)$ $[\therefore \sin\theta = \cos\theta]$

$\therefore 4\mu = 2$

$\therefore \mu = 0.5$

$\therefore N = 10\,\mu = 5$